PA-MDC Grade 4-6 Arrays,Puzzles,and Factor Trees

This lesson is intended to help you asses how well students are able to use “clues” about numbers including factors, multiples, prime, composite, square, even, and odd numbers. In particular this lesson aims to identify students who have difficulties with understanding the difference between primes and composite, or factors and multiples This lesson is adapted from Georgia Department of Education, *Connected Mathematics Prime Time* and *Investigations in Number Data & Space: Mathematical Thinking.*

Concept Development Formative Assessment Lesson

**Arrays, Number Puzzles, and Factor Trees**

**Mathematical goals:** This lesson unit is intended to help you assess how well students are able to use “clues” about numbers including: factors, multiples, prime, composite, square, even, odd, etc. In particular, this unit aims to identify and help students who have difficulties with: understanding the difference between primes and composites, or factors and multiples.

**Common Core State Standards** This lesson involves a range of mathematical practices from the standards, with emphasis on:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

6. Attend to precision.

7. Look for and make use of structure

This lesson asks students to select and apply mathematical content from across the grades, including the content standards:

4-OA: Use the four operations with whole numbers to solve problems.

4-OA: Gain familiarity with factors and multiples.

5-NBT: Understand the place value system.

6-NS: Compute fluently with multi-digit numbers and find common factors and multiples

**PA Core Standards:**

PA CC2.2.4.A.2: Develop and apply number theory concepts to find factors and multiples

PA CC 2.1.5.B.1: Apply place value concepts to show understanding of operations with whole numbers to perform operations including decimals

PA CC.2.1.6.E.3: Develop and or apply number theory concepts to find common factors and multiples.

**Introduction:**  This lesson unit is structured in the following way:

• Before the lesson, students work individually on a pre- assessment that is designed to reveal their current understanding and difficulties. You then review their work, and formulate questions for students to answer, to help them improve their solutions.

• During the lesson, students work in pairs to match the array cards, number clues and factor trees.

• In a whole-class discussion, explain their answers.

• Finally, students return to their original assessment task, and try to improve their own responses.

**Time Needed: estimated 75 – 90 minutes**

* Pre-assessment 15 minutes
* 60 minutes whole lesson – using two sets of cards or a 45 minutes lesson- with one set of cards (using card set two for extension activity)
* Post-assessment 15 minutes

Exact timings will depend on the needs of the class and if both sets of cards are used.

**Materials required:**

* Each individual student will need two copies of the worksheet *Arrays, Number Puzzles, and* *Factor Trees*.
* Each small group of students will need a packet of Card Set 1: Arrays A-J, Number Puzzles 1-8, & Factor Trees a-g; and Card Set 2: Arrays K-T, Number Puzzles 9-16, & Factor Trees h-n copied and cut up before the lesson.
* If students will be cutting the arrays to attach to matching cards –scissors, glue sticks, chart paper and white boards/erasers will be needed.
* Students struggling may use a hundreds board as a tool.

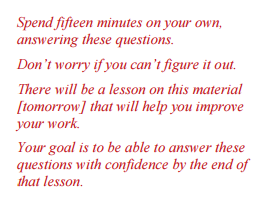
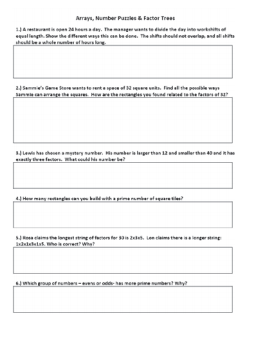
**Before the lesson**

**Assessment task:** Have students do the pre-assessment in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

***Guiding Note: Teachers should take the time to complete the pre-assessment prior to students attempting the task. Using the Formative Assessment Anticipation guide to anticipate where students will show their strengths and their weaknesses. It is also recommended that teachers create their own questions to address the misconceptions uncovered prior to using the questions provided in the guide.***

Give each student a copy of *Arrays, Number Puzzles & Factor Trees.* Introduce the task briefly and help the class to understand the problem and its context.

It is important that students answer the question without assistance, as far as possible. If students are struggling to get started,ask them quesitons that help them understand what is required but do not do the task for them.

  
**Assessing student’s responses :**

Collect students’ responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. The purpose of this is to forewarn you of the issues that will arise during the lesson,so that you may prepare carefully.

We suggest that you do not score students’ work. The research shows that this is counterproductive, as it encourages students to compare scores,and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of theirwork. Some suggestions for these are given on below. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your own students’ work, using the ideas below. You may choose to write questions on each student’s work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

**Common issues - Suggested questions and prompts:**

|  |  |
| --- | --- |
| **Common Issues** | **Suggested questions and prompts** |
| **Student doesn't find all factor pairs for a**  **given number.** | ***How can you make sure you haven't***  ***left out any factor pairs for a number?***  ***How can you use rectangular arrays to decide if you have all the factor pairs?***    ***How can you use a factor tree to decide if you have all the factor pairs?***    ***How could you make a chart to decide if you have all the factor pairs?*** |
| **Lacking conceptual understanding of prime numbers.**  **Student doesn't realize that a prime number**  **will have only one factor pair, only one**  **rectangular array, and it will not have a factor**  **tree if it is prime.**  **Student should also recognize the only EVEN prime number is 2** | ***How many other rectangular arrays can you find for each number?***  ***What do you notice about the number of***  ***rectangular arrays possible for each number?***  ***Which number(s) do not***  ***have a factor tree ? Why not?*** |
| **Student does not realize -One is neither prime nor composite.**  **Students do not understand that you do not use one as a factor in factor trees.** | ***Why don't any of the factor trees include the number 1 as a factor?*** |
| **Not understanding that if there are an odd**  **number of factors for a number, the number is a square number.** | ***Which rectangular arrays are actually squares?***  ***How many total factors do each of your numbers have?***  ***How can you tell if a number will have a square as one of its arrays?*** |
|  |  |

**Suggested Lesson Outline:**

* If you have a limited time scheduled for math you may want to end the lesson after the first collaborative activity.
* Card Set 2 can be used in the exact same way as card set one. Set 2 does include clues using the **exponents** which are not included in set 1. You may use card set 2 for an extensions activity or blank cards for students to create their own arrays, number puzzles and factor trees. Can exchange with other groups or share at the whole class discussion.
* Wrap up the lesson when students are completely satisfied with their own work by together their solution sets that match. They should have eight matching solution sets for Card Set 1 ( and another eight for card set 2)

**Collaborative activity 1: matching card set 1: Arrays A-J, Number Puzzles 1 – 8, & Factor Trees a- g (30 minutes)**

* Organize your class into groups of two. With larger groups, some students may not fully engage in the task. Give each group materials for Card Activity 1.
* Introduce the lesson carefully:

*I want you to work as a team. Take it in turns to match an Array with either a Number Puzzle or a Factor Tree.*

*Each time you create a match, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all of the placements.*

*There is a lot of work to do today, and it doesn’t matter if you don’t all finish. The important thing is to learn something new, so take your time.*

* Your responsibilities as teacher during the small group work are to make a note of various students’ approaches to the task, and to support student problem solving

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For, example, students may know how to find the number represented by each array or factor tree,but have difficulty finding the answer for each number puzzle.

* Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking .

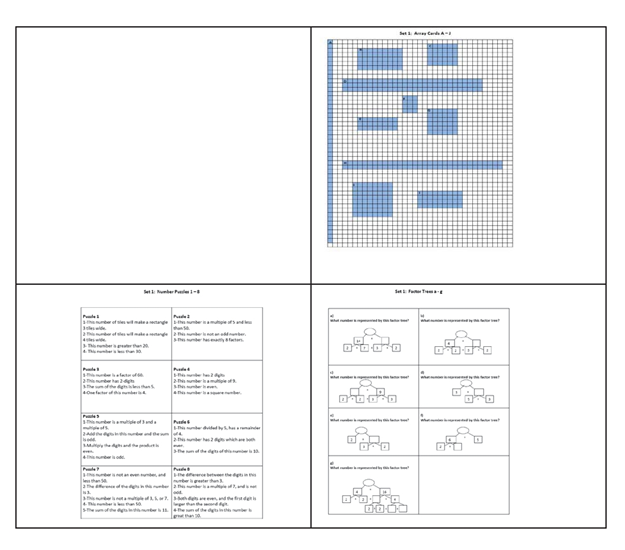
*There is more than one way to tackle this task. Can you think what one of them might be? (finding the number of the array is the most efficient way to find the numbers that will match the puzzles and the trees)*

*Another suggestion may be the use of a hundred board for some students. They can “mark off” numbers that do not fit the criteria for the number puzzles. All of the solutions are for numbers 100 or less*

* If one student has placed a particular card, challenge their partner to provide an explanation

*Gloria placed this Array card with this Number Puzzle. Shane, why has Gloria placed it here?*

* If the whole class is struggling on the same issue, then you may want to write questions on the board and organize a whole class discussion



**Sharing work (10 minutes)**

Students should be instructed to lay their cards on the chart paper but not glue them until directed to do so. When students get as far as they can with matching cards*,* ask one student from each group to visit another group's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

*If you are staying at your desk, be ready to explain the reasons for your group's matches.*

*If you are visiting another group, write your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.*

*If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.*

*When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.*

Students may now want to make changes. They may also glue cards on chart paper.

**Whole Class Discussion: (10minutes)**

Facilitate – this is a student led conversation use your questioning to drive the discussion

Choose students to share how they figured out which method was used most efficiently. If time is limited only choose 2 -3 groups to present.

If any student groups did the extension activity cards have them share some of their solutions.

**Improve individual solutions to the assessment task (10 minutes)**

Return to the students their original assessment, *Arrays, Number Puzzles & Factor Trees with student directed questions*, as well as a second blank copy of the task.

*Look at your original responses and the questions that have been directed to those responses Also think about what you have learned this lesson.*

*Using what you have learned, try to improve your work.*

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

**Answer Key Set 1**

|  |  |  |  |
| --- | --- | --- | --- |
| **Array Card** | **Number Puzzle** | **Factor Tree** | **Number** |
| **E** | **1** | **b** | **24** |
| **C** | **2** | **f** | **30** |
| **F** | **3** | **e** | **12** |
| **G.J** | **4** | **c** | **36** |
| **B** | **5** | **d** | **45** |
| **H,I** | **6** | **g** | **64** |
| **A** | **7** | **--** | **47** |
| **D** | **8** | **a** | **84** |

**Answer Key Set 2**

|  |  |  |  |
| --- | --- | --- | --- |
| **Array Card** | **Number Puzzle** | **Factor Tree** | **Number** |
| **R** | **9** | **k** | **70** |
| **D,T** | **10** | **l** | **28** |
| **K** | **11** | **--** | **17** |
| **N,P** | **12** | **m** | **20** |
| **S** | **13** | **h** | **50** |
| **L** | **14** | **n** | **16** |
| **O** | **15** | **l** | **100** |
| **M** | **16** | **j** | **75** |

**Pre- Assessment Task Solutions :**

**For questions 1 and 2 student responses will vary depending on how they solve multiplication**

**problems. Be sure that the responses have correct interpretations of each model.**

**Question 1:**

**# of shifts 1 2 3 4 6 8 12 24**

**# of hours** **24 12 8 6 4 3 2 1**

**Question 2: The dimensions of the sides of the rectangles should be the factors for 32. Students should draw rectangles with the following dimensions:**

**1 X 32, 2 X 16, 4 x8 (or the reverse)**

**Questions 3: Numbers with exactly three factors would be square numbers. Square numbers between 12 and 40 are 16, 25, & 36. Only 25 has three factors.**

**Question 4: Only one because prime numbers only have two factors – one and itself.**

**Question 5: Rosa is correct. Strings of factors at the bottom of a factor tree do not include 1**

**Question 6: All prime numbers are odd, except for the number 2 so ODDS have more prime numbers.**

**Student Name**

**Arrays, Number Puzzles & Factor Trees**

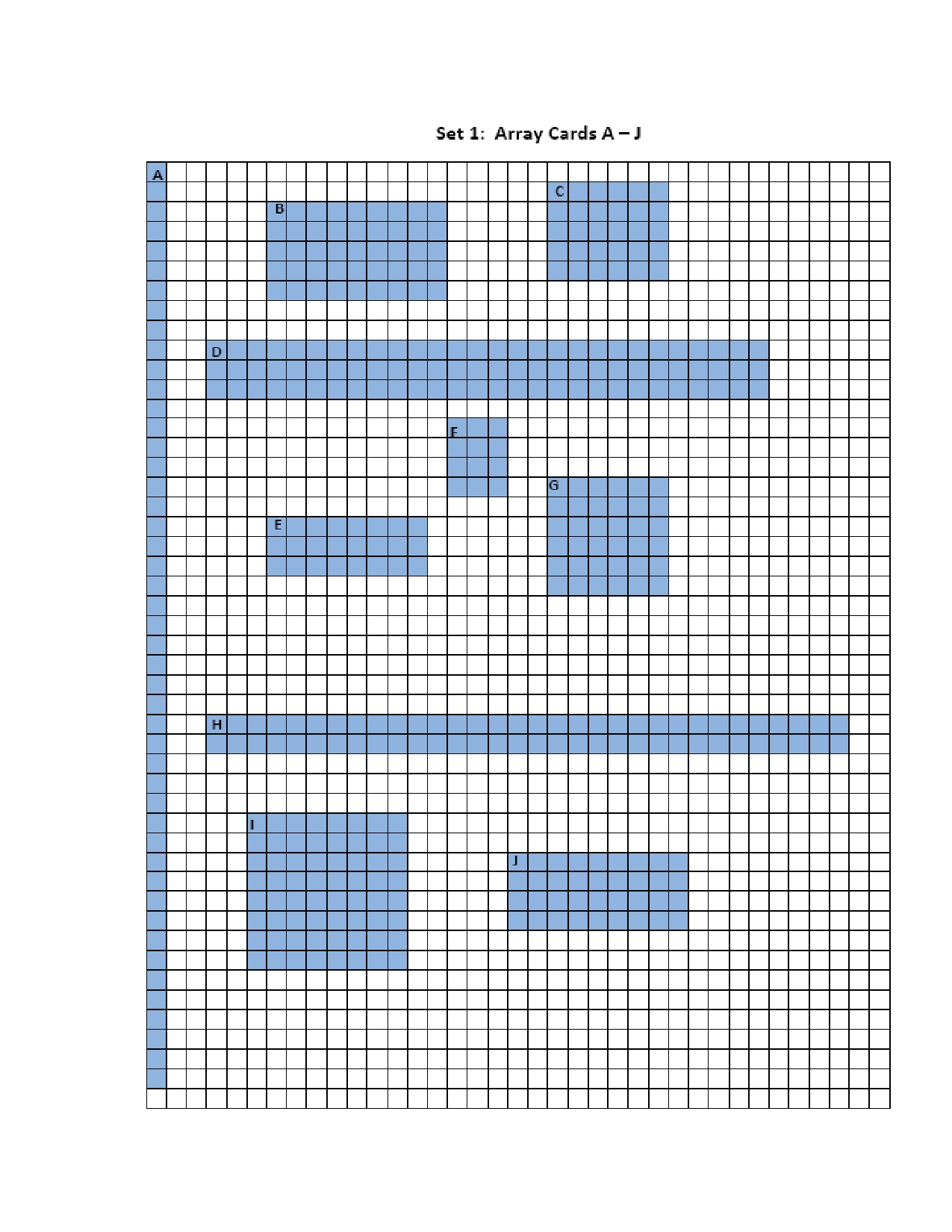
1. A restaurant is open 24 hours a day. The manager wants to divide the day into work- shifts of equal length. Show the different ways this can be done. The shifts should not overlap, and all shifts should be a whole number of hours long.
2. Sammie's Game Store wants to rent a space of 32 square units. Find all the possible ways Sammie can arrange the squares. How are the rectangles you found related to the factors of 32?
3. Lewis has chosen a mystery number. His number is larger than 12 and smaller than 40 and it has exactly three factors. What could his number be?
4. How many rectangles can you build with a prime number of square tiles?

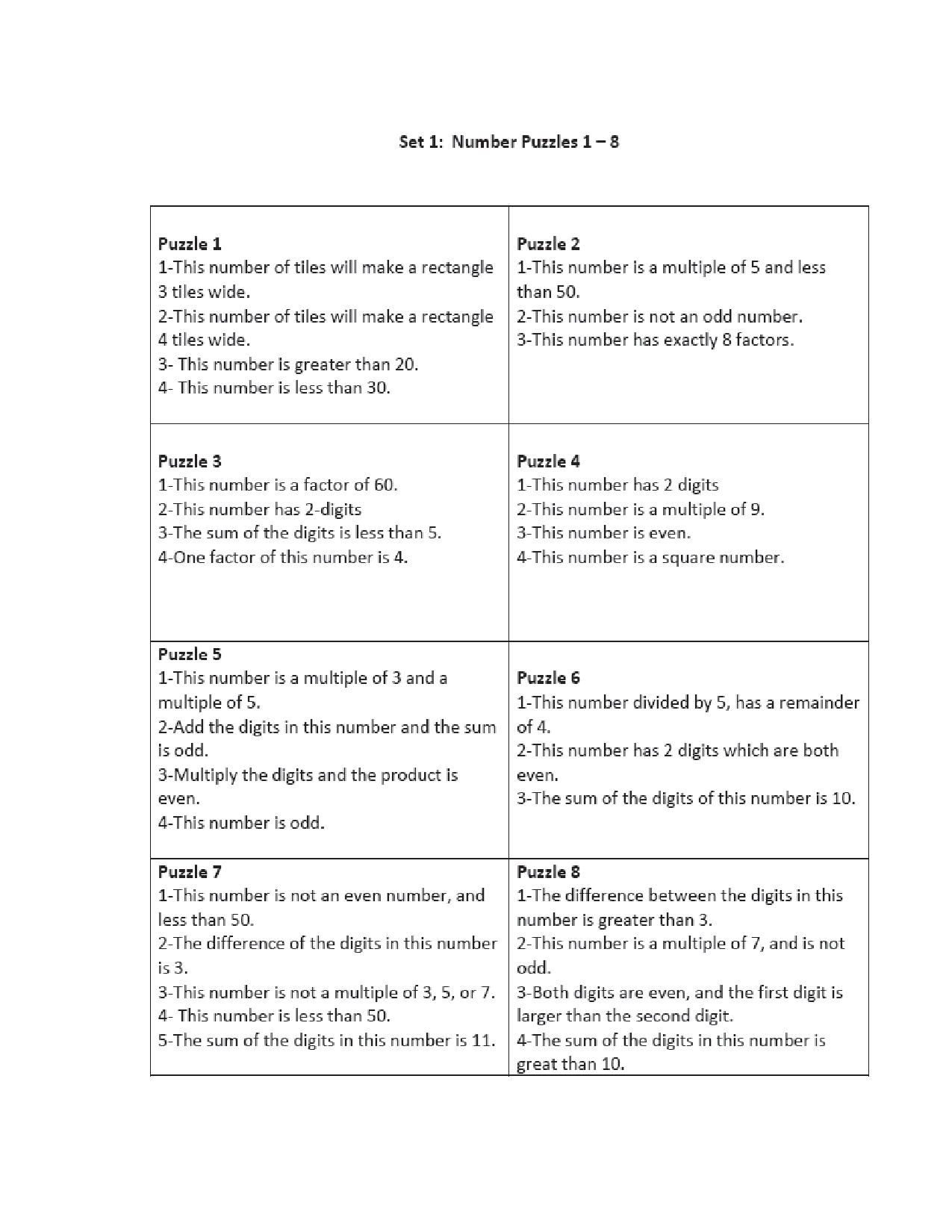
5) Rosa claims the longest string of factors at the bottom of a factor tree for 30 is 2 x3x5.

Lon claims there is a longer string for the factor tree : 1x2x3x1x5, Who is correct ?Why?

6) Which group of numbers – even or odd – has more prime numbers? Why?

`





|  |  |
| --- | --- |
| 1. ***What number is represented by this factor tree?***   **14**4  ***x***  **2**  **2**  ***x x x x***  **3**  **7** | 1. ***What number is represented by this factor tree?***   **4**  ***x***    **2**  **2**  ***x x x***  **3**  **2** |
| **c) What number is represented by this factor tree?**    **9**  **4**  **X**  **3**  **X X X**  **2**  **3**  **2** | **d) What number is represented by this factor tree?**  **X**  **3**  **5**  **3**  **X** |
| **e)What number is represented by this factor tree?**  **2**  **X**  **2**  **3**  **X** | **f) What number is represented by this factor tree?**    **5**  **6**  **X**  **2**  **X** |
| ***g) What number is represented by this factor tree?***  **4**  **16**  ***X***  **4**  **2**  **2**  ***X X***    **2**  **2**  ***X X X*** |  |

**Set 2 Factor Trees, h – n**

|  |  |
| --- | --- |
| **h) What number is represented by this factor tree?**  **5**  **X**  **2**  **5**  **X** | **i) What number is represented by this factor?**  **10**  **X**    **2**  **5**  **2**  **5**  **X X X** |
| **j) Wat number is represented by this factor tree?**  **5**  **X**    **3**  **5**  **X** | **k) What is the number represented by this factor tree?**  **2**  **14**  **5**  **X**  **X** |
| **l) What number is represented by this factor tree?**  **7**  **X**  **2**  **2**  **X** | **m) What number is represented by this factor tree?**  **2**  **X**  **2**  **5** |
| **n) What number is represented by this factor treve?**  **4**  **8**  **2**  **X**  **X**  **2**  X **X** |  |

